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Topological and Geometric Tools for the Analysis fo Complex Networks

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**Final Report on "Topological and Geometric Tools for the Analysis fo Complex Networks" Ali Jadbabaie, Victor Preciado, University of Pennsylvania
Fan Chung Graham, UCSD
Shing-Tung Yao, Harvard University**

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Background and Introduction

Many current military as well as civilian technical visions convincingly suggest that net-centric technology can provide unprecedented levels of performance, robustness, and efficiency. Unfortunately, there is substantial confusion regarding the obstacles to achieving this vision, and our proposed program over the past few years has gone a long way towards clarifying and address the central research challenges in the area of complex networked systems.

Essentially what is needed is a deeper understanding of network structure and function, beyond naive graph-theoretic measures of large-scale connectivity that incorporates the domain-specific drivers and constraints on system organization and dynamics.

Our work over the period of the award (June 2009-October 2013) has focused on developing a rigorous understanding of the applications of geometric and topological tools for study and analysis of complex networks.

Our advances in these areas have been focused along 2 major thrusts:

1. New algorithmic paradigm for analysis of networks
2. New tools for analysis of networks beyond graphs.

Each thrust was divided into a collection of major tasks, which will be described below:
For thrust 1, we completed the following tasks

1. We developed a new paradigm for analysis of networks represented by massive graphs. Our focus was on new approaches for analysis of local algorithms for graph partitioning that utilizes our pioneering work on spectral graph theory, diffusions, random walks and Markov chains. Specifically, we developed tools and algorithms whose complexity does not scale with the size of the network. This was based on approximately solving for PageRank , Discrete Poisson Equations as well as distributed solution Poisson equations.

2. Motivated on the algorithms developed in part I, we developed scalable, second order methods for solving network utility maximization problems using distributed implementation of Newton Methods

3. We developed and extended notions of Ricci curvature from manifolds to graphs

4. We defined the notion of quantum tunneling on graphs

5. We developed geographic routing strategies for networks, using Ricci curvatures and conformal maps.

6. We developed tools for analysis of large scale complex networks based on local structural properties. Specifically we related behavior of dynamic processes on graphs to eigenvaleus of the laplacian and adjacency matrices and showed how local structural properties such as subgraph frequencies relate to moments on distribution of eigenvalues

In the second thrust, we showed how tools from algebraic topology can be utilized to generalize the notion of random walks and harmonic functions to simplicial complexes, discrete structures that encode highdr order relations beyond binary that re represented by graphs. Specifically, we showed how homology of simplicial complexes can be characterized in distributed ways by solving Laplacian equations on complexes. We further generalized the notion of Page Rank, w a popular algorithm for ranking nodes based on their improtance beyond graphs to simplicial complexes. This allows us to develop ranking schemes for higher order cells such as edges and hyper edges. Moreover, we showed how vectorized Laplacians can be utilized to develop rankings for edges and higher order cells and how this can lead to sparsification algorithms.

More importantly, our project lead to building significant bridges between seemingly disparate areas: discrete mathematics and graph theory, geometry, optimization, algebraic topology, distributed computation, and approximation algorithms

Progress Report : Some highlights

I. Algorithmic Advances in Network Science
Primary PI Interactions: Chung, Jadbabaie, Preciado

PIs Jadbabaie and Chung have also continued their basic algorithmic and theory work on largescale networks. There has been notable collaboration between the two groups, particularly on the topic of local algorithms, described below. PIs Jadbabaie and Chang have focused on developing a deep understanding of geometric and topological analysis methods on networks. This has lead to insights on achieving dramatic speed-ups in distributed algorithms for network optimization problems which PI Jadbabaie and his group have developed over the past year.

In particular, PIs Jadbabaie and Chung are focusing on using graph sparsification techniques developed by Chung to create distributed Newton and Quasi-Newton type algorithms for network optimization that are orders of magnitude faster than the current subgradient techniques for network optimization. Below, we will highlight some of the major advances. Some highlights are described below:

Vectorized Laplacian in connection graphs

Connection graphs are generalizations of weighted graphs in which each edge is associated with both a positive scalar weight and a d-dimensional rotation matrix for some fixed positive integer d. The Laplacian of a connection graphs are higher dimensional versions of the normalized Laplacian matrix, which are linear operators acting on the space of vector-valued functions (instead of the usual real-valued functions). For high dimensional data sets, a central problem is to uncover lower dimensional structures in spite of possible errors or noises.

An approach for reducing the effect of errors is to consider the notion of inconsistency, which quantifies the difference of accumulated rotations while traveling along distinct paths between two vertices. In many applications, it is desirable to identify edges causing the inconsistencies, or to identify portions of the graph that have relatively small inconsistency. We give an algorithm is given, utilizing a version of effective resistance from electrical network theory, that deletes edges of a connection graph in such a way that reduces inconsistencies. In this paper, rather than deleting edges, our focus is on identifying subsets of a connection graph with small inconsistency. The notion of consistency of a subset of the vertex set of a connection graph will be introduced, which quantifies the amount of inconsistency for the subset to within an appropriate error bound.

A Distributed Newton Method for Network Flow Optimization and Utility Maximization

Most existing work uses dual decomposition and subgradient methods to solve network optimization problems in a distributed manner, which suffer from slow convergence rate properties. In this collection of papers we have proposed an alternative distributed approach based on a Newton-type method for solving minimum cost network optimization problems and utility maximization problems. The key component of the method is to represent the dual Newton direction as the solution of a discrete Poisson equation involving the graph Laplacian. This representation enables using an iterative consensus-based local averaging scheme (with an additional input term) for the network flow problem and a more sophisticated matrix splitting technique to compute the Newton direction based only on local information. We show that even when the iterative schemes used for computing the Newton direction and the stepsize in our method are truncated, the resulting iterates converge superlinearly within an explicitly characterized error neighborhood. Simulation results illustrate the significant multiple order of magnitude performance gains of this method relative to subgradient methods based on dual decomposition.

Effects of delay, nonlinearity, stochasticity, and network structure on the functionality and optimal control of large scale networks

The coordinated motion of multi-agent systems (such as flocking and consensus) and oscillator synchronization are two important examples of networked dynamical systems. In this body of work which highlights our efforts on studying the interplay of network dynamics, randomness, form and function, we consider what effect multiple, noncommensurate (heterogeneous) communication delays can have on the consensus properties of large-scale multi-agent systems

endowed with nonlinear dynamics. We show that the structure of the delayed dynamics allows functionality to be retained for arbitrary communication delays, even for switching topologies under certain connectivity conditions. The results are extended to the related problem of oscillator synchronization.

A. Papachristodoulou and A. Jadbabaie, Delay Robustness of Nonlinear Internet Congestion Control Schemes" *IEEE Transactions on Automatic Control,* Vol. 55, No. 5, May 2010

A. Papachristodoulou and A. Jadbabaie, and U. Munz, Effects of Delay in Multi-Agent Consensus and Oscillator Synchronization, *IEEE Transactions on Automatic Control,* Vol. 55, No. 6, June 2010

A. Tahbaz-Salehi, and A. Jadbabaie," Consensus in ergodic-stationary graph processes," IEEE Transactions on Automatic Control, Vol. 55, No. 1, pp. 225 - 230, January 2010

PageRank Algorithm, local algorithms and graph sparsification
Fan Chung, Ali Jadbabaie

PageRank, which was first introduced by Brin and Page, is at the heart of Google's web searching algorithms. Originally, PageRank was designed for the Web graph, but for any given graph, PageRank is well-defined and can be used for capturing quantitative correlations between pairs of vertices as well as pairs of subsets of vertices. In addition, PageRank vectors can be efficiently computed and approximated. For many applications, it is quite crucial to have a sharper error bound for PageRank. We give an improved algorithm for computing personalized PageRank vectors with tight error bounds; the improved PageRank algorithm is crucial in computing a quantitative ranking for edges in a given graph. The edge ranking can then be used to examine two interrelated problems: graph sparsification and graph partitioning. We then combine the graph sparsification and the partitioning algorithms using PageRank vectors to derive an improved partitioning algorithm..

We have utilized the graph sparsification techniques for fast computation of Newton steps in network optimization problems discussed in the first part of this section. Furthermore, we are currently under

From Local Measurements to Network Spectral Properties: A formal theory of Network Science
Victor Preciado and Ali Jadbabaie

It is well-known that the behavior of many dynamical processes running on networks is intimately related to the eigenvalues of the graph Laplacian of the underlying network. We have addressed the problem of inferring global information regarding the eigenvalue spectrum of a network from a set of local samples of its structure. In particular, we have found explicit relationships between the so-called spectral moments of a graph and the presence of certain small subgraphs, also called motifs, in the network. Since the eigenvalues of the network have a direct influence on the network dynamical behavior, our result builds a bridge between local network measurements (i.e., the presence of small subgraphs) and global dynamical behavior (via the spectral moments). Furthermore, based on our result, we have proposed novel decentralized algorithm to compute the spectral moments of a network by aggregating local measurements of the network topology. Our final objective is to understand the relationships between the behavior of dynamical processes taking place in a large-scale complex network and its local topological properties.

Victor M. Preciado, A. Tahbaz-Salehi, and A. Jadbabaie, “On Asymptotic Consensus Value in Directed Random Networks,” *IEEE Conference on Decision and Control*, 2010.

V.M. Preciado , A. Tehbaz-Salehi and A. Jadbabaie, “Variance Analysis of Randomized Consensus in Switching Directed Networks,” *American Control Conference*, 2010.

V. Preciado A. Tahbaz-Salehi, and A. Jadbabaie, " Variance analysis of consensus in directed switching random networks," *Proceedings of the 2010 American Control Conference*, to appear

V. Preciado M. Zavlanos, A. Jadbabaie, and G. J. Pappas " Distributed Control of the Laplacian Spectral Moments of Networks," *Proceedings of the 2010 American Control Conference*, to appear

V.M. Preciado and A. Jadbabaie, “Spectral Analysis of Virus Spreading in Random Geometric Networks,” *IEEE Conference on Decision and Control*, 2009.

V.M. Preciado and A. Jadbabaie, “Moment-Based Analysis of Synchronization in Small-World Networks of Nonlinear Oscillators,” *IEEE Conference on Decision and Control*, 2009.

Social networks: structure, and information aggregation

Finally, we have developed a dynamic model of opinion formation in social networks in which relevant information is spread throughout the network in such a way that no agent has enough data to learn a payoff-relevant parameter. Individuals engage in communication with their neighbors in order to learn from their experiences. However, instead of incorporating the views of their neighbors in a fully Bayesian manner, agents use a simple updating rule which linearly combines their personal experience and the views of their neighbors (even though the neighbors' views may be quite inaccurate). This non-Bayesian learning rule is motivated by the formidable

complexity required to fully implement Bayesian updating in networks. We show that, under mild assumptions, repeated interactions lead agents to successfully aggregate information and to learn the true underlying state of the world. This result holds in spite of the apparent na¨ıvit´e of agents' updating rule, the agents' need for information from sources (i.e., other agents) the existence of which they may not be aware of, the possibility that the most persuasive agents in the network are precisely those least informed and with worst prior views, and the assumption that no agent can tell whether their own views or their neighbors' views are more accurate.

A. Tahbaz-Salehi, A. Sandroni, and A. Jadbabaie "Non-Bayseian Social learning," Games and Economic Behavior, 2012

A. Tahbaz-Salehi, A. Sandroni, and A. Jadbabaie " Social learning under peer pressure" *Proceedings of the IEEE Conference on Decision and Control 2009*, pp, 4446-445 (**Finalist, Best paper award***)*

A. Tahbaz-Salehi, "Complex Networks: New Models and Distributed Algorithms,", *PhD thesis,* Department of Electrical and Systems Engineering, University of Pennsylvania, (**Winner of the best thesis prize)**

Trust and distrust in ranking algorithms: From Diffusion to Dirichlet Problems
Fan Chung

The idea of ranking nodes in a graph has a rich history starting from the introduction of PageRank by Brin and Page. The original PageRank was meant for Web Search, but many researchers have developed more tailored rankingsystems such as personalized PageRank, giving a ranking relative to some specified starting distribution ~s. One pitfall with PageRank as a ranking system is the fact that all edges contribute positively. In practice, an edge such as a link from one Web page to another can also represent a negative interaction or distrust between the nodes.

Motivated by the continual development of new PageRank-based algorithms and analysis of Dirichlet eigenvectors, we develop and analyze Dirichlet PageRank algorithms. For a connected graph G, we examine a Dirichlet PageRank equation and compute the unique solution with Dirichlet boundary conditions pr(v) = 0 for vertices v on the boundary of a specified vertex subset S. Furthermore we generalize the boundary conditions to arbitrary values pr(v) = sigma(v) for boundary vertices v. We give an efficient algorithm ApproxDirichPR to compute approximate Dirichlet PageRank vectors with any boundary condition
Several applications of Dirichlet PageRank with boundary conditions includes: a method to (a) diminish known spammers' influence, (b) adjustments for trusted friends' opinions, (c) validating rank for newly-created nodes, (d) reconciling ranking in personal and global social networks.

Resource allocation in dynamic networks
Efficient allocation of resources to meet every changing demand is a task arising in numerous applications. For example, various institutions, such as governments or corporations, respond to the needs of a populace and wish to meet demand within allowed expenditure of resources. In the case where demand spreads, one has to be able to act precisely to act before demand becomes unmanageable. For example, if the demand is the need for medicine being spread by an epidemic, then an institution wants a way to distribute antidote to contain the disease.

We have studied a variant of the classical contact process, a continuous time Markov process which is often used to model the spread of disease. In our scenario, instead of disease we have multiple types of demands which grow, spread and interact. For instance, the demand for iPhones may influence the demand for iPads. Demand might reasonably be considered discontent with current supply and discontent naturally spreads. In our model, initially, a subset of the vertices is unsatisfied, with demands for certain goods. Demands then spread and grow over time according to specified rules. Our goal is to identify how often to ship to each vertex to contain and satisfy demands, given an initial seed set.

Our analysis is comprised of two parts. First we give conditions which ensure that all demand is satisfied in O(log n) time, with high probability, regardless of the initial demand. This is a global solution, in this sense that it involves scheduling shipments to all vertices in the graph, in a way that will be made precise once the model is formally introduced. Then we analyze a situation where shipments are scheduled to only a subset of a vertices containing and influenced by the initial demand. In particular, we are interested in subsets so that the demand within the subset is satisfied quickly (in O(log n) time) and the demand for vertices not receiving shipments are with very low probability.
Our analysis allows a tradeoff: On one hand, we would like to guarantee that the demand escapes a set with probability at most _. Our results will allow us to use PageRank (or a sharper Kronecker PageRank introduced to identify a subset of vertices and to determine the supply rates that yields such a guarantee. On the other hand, if we would like to send shipments to a particular set of vertices, then our analysis allows a guaranteed bound on the escape probability. In summary, we consider an interactive variant of the contact process concerning multi-commodity allocation. In this process, the demands for several types of commodities are initially given at specified vertices and then the demands evolve/spread interactively in the contact graph according to rules which can be described by a spread matrix. To allocate supplies in such a dynamic setting, we use a modified version of PageRank vectors, called Kronecker PageRank, to give an allocation scheme. We analyze both the situations that the evolving demand distribution form clusters around the initial demand and the case that the demands spread to the whole network. We analyze and establish sharp upper bounds for the probability that the demand unsatisfied as a function of PageRank vectors.

Games on Graphs

We have studied various dynamic phenomena on graphs in the past, including strategic models where networks emerge as an equilibrium of strategic interaction between agents and non-strategic stochastic dynamic network formation and opinion aggregation models where agents in a social network exchange opinions as they build friendship ties and the more they meet the more they influence each other's opinion.

Optimization-based control of epidemics on complex networks

We have studied the problem of containing spreading processes in arbitrary directed networks by distributing protection resources throughout the nodes of the network. We consider two types of protection resources are available: (i) Preventive resources able to defend nodes against the spreading (such as vaccines in a viral infection process), and (ii) corrective resources able to neutralize the spreading after it has reached a node (such as antidotes). We assume that both preventive and corrective resources have an associated cost and study the problem of finding the cost-optimal distribution of resources throughout the nodes of the network. We analyze these questions in the context of viral spreading processes in directed networks. We study the following two problems: (i) Given a fixed budget, find the optimal allocation of preventive and corrective resources in the network to achieve the highest level of containment, and (ii) when a budget is not specified, find the minimum budget required to control the spreading process. We show that both resource allocation problems can be solved in polynomial time using Geometric Programming (GP) for arbitrary directed graphs of non-identical nodes and a wide class of cost functions. Furthermore, our approach allows one to optimize simultaneously over both preventive and corrective resources, even in the case of cost functions being node- dependent. We illustrate our approach by designing optimal protection strategies to contain an epidemic outbreak that propagates through an air transportation network.

2. Beyond graphs: towards a network theory of simplicial complexes

PIs: Jadbabaie, and Chung, senior personnel: G Lippner(Harvard)

Use of Cohomology Theory in Estimation

We have studied the problem of estimating the state of sensors in a sensor network from noisy pairwise relative measurements. The underlying sensor network is typically modeled by a graph whose edges correspond to pairwise relative measurements and nodes represent sensors. Using tools from algebraic topology and cohomology theory, we present a new model in which the higher order relations between measure- ments are captured as simplicial complexes. This allows us to address the fundamental tension between two conflicting goals: finding estimates that are close to obtained measurements, and at the same time are consistent around any sequence of pairwise measurements that form a cycle. By defining a measure of inconsistency around each cycle, we present a one-parameter family of algorithms that solves the estimation problem by identifying and removing the smallest fraction of measurements that make the estimates globally inconsistent.

We demonstrate that the inconsistencies are due to topological obstructions and can be decomposed into local and global components that have interesting geometric interpretations. Furthermore, we show that the proposed algorithm is naturally distributed and will provably result in consistent estimates, and more importantly, recovers two sparse estimation algorithms as special cases.

From graphs to simplicial complexes : **Lapalcian Flows and Dirichlet Problems**
Jadbabaie, Lippner

We have created a new theory of network analysis by generalizing concepts that are well studied on graphs (random walks, Dirichlet problems, Page Rank) and study them in the context of random and deterministic simplicial and other chain complexes. As part of this program, we used discrete Hodge Theory to define combinatorial Laplacian operators over simplicial complexes. We then utilized Laplacian flows as a basis of distributed algorithms to compute homology and cohomology over the Reals, allowing to answer questions related to distributed estimation, sensor network coverage, and so forth. The main question of random walks on simplicial complexes, what the walk would mean and how it should be structured is an important open question that was answered in this project. In particular, whether there is a notion of mixing-time, and how eigenvalues of the combinatorial Laplacian might quantify the mixing. This was characterized in a recent collaboration between Jadbabaie and the Harvard Team. In particular, Chung and Jadbabaie have leveraged our results on Discrete Dirichlet problems and how they might generalize to simplicial complexes. One immediate application of such a result would be on defining page-rank style centrality measures for higher order cells, such as edges and faces.

3. Other highlights

Plenary Talks

PI Jadbabaie gave plenary talks at the inaugural IFAC Conference on Network Control Systems in September 2009, in the IEEE Conference on Decision and Contorl, in December 2009, and in the Belgian Bi-annual Conference on Dynamical Systems, Control and Optimization in June 2010. HE also gave plenary talks in the Network Measurement and Mapping Conference in summer of 2013, and in Network Economcis and Game Theory Workshop in 2013

Awards and Honors: Jadbabaie became an IEEE Fellow, and the Alfred Fitler Moore Professor of Network Science at University of Pennsylvania.

Project-Related Synergistic Activities on Education

PI Jadbabaie has co-founded a new undergraduate program with the project research themes at its core. The program, named the Singh Program in Network and Social Systems at Penn Engineering (NETS), is an interdisciplinary program in the School of Engineering and Applied Sciences that will train students in network science and social systems that are changing the way we make decisions, create and exchange products in an increasingly networked world. The program draws heavily from network science, algorithmic game theory, dynamics, distributed decision making systems as well as optimization techniques while emphasizing design of technology and engineering for users and organizations in emerging social systems and markets. Essentially, the goral of the program is to produce graduates who would be well suited to do research in the areas at the core of this project.

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Abstract

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1. V. Preciado, A. Jadbabaie, G. C. Verghese, " Structural analysis of Laplacian spectral properties with application to electric transmission networks," IEEE Transactions on Automatic Control, Vol. 58, No. 9, pp. 2338-2343, September 2013
2. V. Preciado and A. Jadbabaie," Moment-based analysis of spreading processes from network structural information," to appear in IEEE Transactions on Networking, Vol. 21, No. 2, pp. 373-382, April 2013

3. E. Wei, A. Ozdaglar, and A. Jadbabaie, "A distributed Newton Method for networkutility maximization,Part I" IEEE Transactions on Automatic Control, Vol. 58, No. 9, pp. 2162-2176, September 2013

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Changes in research objectives (if any):

added a new thrust in distribute doptimzation

Change in AFOSR Program Manager, if any:

None

Extensions granted or milestones slipped, if any:

None

AFOSR LRIR Number

LRIR Title

Reporting Period

Laboratory Task Manager

Program Officer

Research Objectives

Technical Summary

Funding Summary by Cost Category (by FY, $K)

	Starting FY	FY+1	FY+2
Non-Military Government Personnel Costs			
In-house Contractor Costs			
Travel (Be Specific)			
Training (Be Specific)			
Supplies			
Other Expenses (Be Specific)			
Total Resource Requirements			

Report Document

Appendix Documents

2. Thank You

E-mail user

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User Agent:	Mozilla/5.0 (Macintosh; Intel Mac OS X 10_6_8) AppleWebKit/537.36 (KHTML, like Gecko) Chrome/31.0.1650.63 Safari/537.36
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